J Intell Robot Syst (2007) 49:237–251
DOI 10.1007/s10846-007-9136-y

Coprime-factorized Model Predictive Control for Unstable Processes with Delay

Igor Škrjanc

Received: 6 September 2006 / Accepted: 21 January 2007 /
Published online: 6 March 2007

Abstract This paper presents coprime-factorized model predictive control. The main idea of the proposed approach is in process-output prediction based on a coprime-factorized process model. The proposed approach provides a framework to design the control for a wide range of processes such as: higher order, phase non-minimal, unstable and also multivariable. In the paper the coprime-factorized predictive design methodology was studied and implemented on unstable processes with a time-delay which are very difficult to control. The proposed methodology leads to a simple analytical control law, which results in much better performance than previously known control methods.

Key words predictive control · unstable processes

1 Introduction

In recent years a lot of effort has been put into the control of unstable processes with a time delay. This is due to the fact that a lot of processes, especially in the chemical industry, are unstable, and it seems that the problem was not satisfactorily solved. Research was focused on PID tuning rules, which provides acceptable performance together with a simple structure. In [2] and [16] two different tuning methods are proposed. However, they result in poor closed-loop performance, an excessive overshot and a long settling time. A better approach was proposed by [13] and [7] where they present improved PID controller tuning. The most promising results using the PID structure were shown by [11] using an enhanced PID control strategy. The PID parameters for this type of process are also tuned in the frequency domain, as proposed by [14].

I. Škrjanc (✉)
Faculty of Electrical Engineering, University of Ljubljana,
Tržaška 25, 1000 Ljubljana, Slovenia
e-mail: igor.skrjanc@fe.uni-lj.si

Another way to approach the control of unstable processes with a time delay is the internal model control (IMC) strategy [15], [5], [6]. The IMC approach can offer a performance that is comparable to that using the enhanced PID control reported by [12]. The weakness of this approach is the design, which is based on the choice of an appropriate damping factor and a suitable settling time; this can only be done by trial and error. The model predictive formulation for the control of open-loop unstable processes has also been reported as an optimization control problem [10]. This could be very time consuming and unusable in the case of unstable processes with a time delay.

The main idea of the approach presented here is the prediction of the process output, which is based on the coprime-factorized process model. The control algorithm is called a coprime-factorized model predictive functional control (CFMPC). This approach gives a framework for designing the control for a wide range of different processes: unstable, phase non-minimal, processes of higher order and multi-variable processes. The proposed control algorithm was implemented in the case of unstable processes with a time delay, which are one of the most difficult control problems. The problem of constraints is not discussed here. This problem can, in the case of inner model-based approaches, be successfully solved by implementing the constraints into the inner process model, which is part of the control algorithm. To show the potential of the proposed approach, a comparison was made with the results reported by [12], which seem to be the most promising known in the literature. The approach presented here offers a very simple design, which results in higher performance, as proposed by [12], especially in the case of reference tracking, and very similar performance in the case of disturbance rejection.

The paper is organized in the following way: in Section 2 the coprime-factorized model predictive control is introduced; in Section 3 the robust stability of CFMPC is discussed; in Section 4 the simulation study is given, where the comparison with the examples known from the literature [12] is presented.

2 Coprime-factorized Model Predictive Control

In this section the coprime-factorized model predictive functional control will be introduced. Let as assume a single-input single-output process plant described in z-domain as $G_p(z)$. The plant model is defined as $G_m(z)$, and the uncertainty between the actual plant and the model is given by an additive unstructured uncertainty

$$G_p(z) = G_m(z) + \Delta(z) \tag{1}$$

The uncertainty $\Delta(z)$ is unknown but stable. Let us introduce the process model as $G_m(z) = G_m^0(z)z^{-d}$, where $G_m^0(z)$ stands for the dynamical part of the model without delay and z^{-d} defines the pure time delay of the process model. The output of the undelayed process model is denoted as $y_m^0(k)$ and the delayed process model output is denoted as $y_m(k)$. The dynamical part of the process-model transfer function $G_m^0(z)$ has, in general, r unstable poles and will be described by the following discrete-time transfer function

$$G_m^0(z) = \frac{B(z)}{A^-(z)A^+(z)} \tag{2}$$

where $A^{-}(z)$ stands for a polynomial with stable poles and $A^{+}(z)$ stands for a polynomial with r zeros outside the unit circle. It is denoted as

$$A^{+}(z) = z^{r} + a_{1}^{+} z^{r-1} + \cdots + a_{r-1}^{+} z + a_{r}^{+} \tag{3}$$

It is always possible to find the coprime factorization of certain transfer functions, i.e., it is always possible to find two interconnected transfer functions with no unstable pole-zero cancelation [4], [8]. This means that it is possible to factorize the unstable part of the process transfer function to the feedback interconnection of two stable systems, as given in Eq. 4.

$$\frac{1}{A^{+}(z)} = \frac{1}{\mathcal{A}(z)} \frac{1}{1 - \frac{\mathcal{B}(z)}{\mathcal{A}(z)}} \tag{4}$$

where $\mathcal{A}(z)$ stands for a stable polynomial defined by the designer. The poles of the polynomial

$$\mathcal{A}(z) = z^{r} + \alpha_{1} z^{r-1} + \cdots + \alpha_{r-1} z + \alpha_{r}$$

affect the performance and the robustness of the control system. The nominator of the feedback transfer function equals

$$\mathcal{B}(z) = \beta_{1} z^{r-1} + \cdots + \beta_{r-1} z + \beta_{r}$$

where the coefficients of $\mathcal{B}(z)$ are calculated to fulfill Eq. 4. This means that the coefficient of the polynomial $\mathcal{B}(z)$ should equal

$$\beta_{i} = \alpha_{i} - a_{i}^{+}, \; i = 1, \ldots, r \tag{5}$$

Figure 1 shows the coprime-factorization of the unstable transfer function $G_{m}^{0}(z)$ (Eq. 2) of the process model. Using Eq. 2 and the proposed coprime-factorization, y_{m}^{0} is expressed in z-domain in the following way

$$y_{m}^{0}(z) = \frac{B(z)}{A^{-}(z)\mathcal{A}(z)} u(z) + \frac{\mathcal{B}(z)}{\mathcal{A}(z)} y_{m}^{0}(z) \tag{6}$$

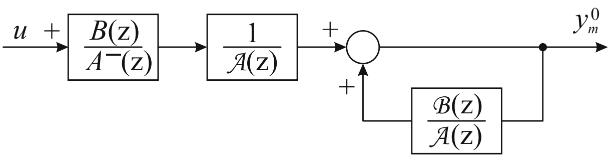
Assuming the ideal model of the process ($y_{m}^{0} = y_{p}^{0}$), Eq. 6 becomes as follows:

$$y_{m}^{0}(z) = \frac{B(z)}{A^{-}(z)\mathcal{A}(z)} u(z) + \frac{\mathcal{B}(z)}{\mathcal{A}(z)} y_{p}^{0}(z) \tag{7}$$

where $y_{p}^{0}(k)$ stands for the undelayed process output, which cannot be measured directly, but it can be estimated, using the delayed process model output $y_{m}(k)$ and the undelayed process model output $y_{m}^{0}(k)$, as will be shown later. The transfer functions $\frac{B(z)}{A^{-}(z)\mathcal{A}(z)}$ and $\frac{\mathcal{B}(z)}{\mathcal{A}(z)}$ will be further denoted as $G_{m_1}(z)$ and $G_{m_2}(z)$.

The prediction of the model output, which is necessary to obtain the predictive control law, is calculated in the state-space domain. Under assumption of process

Fig. 1 Coprime factorization of dynamical part ($G_{m}^{0}(z)$) of the process model



plant observability, both transfer functions, $G_{m_1}(z)$ and $G_{m_2}(z)$, can be transformed in to the observable canonical state-space form (the detailed explanation is given in Appendix). The relation between the transfer functions and the state-space representation is the following:

$$G_{m_i}(z) = \mathbf{C}_{m_i}\left(z\mathbf{I} - \mathbf{A}_{m_i}\right)^{-1}\mathbf{B}_{m_i},\ i = 1, 2 \tag{8}$$

Each transfer function is, in state-space domain, presented by the triple $\mathbf{A}_{m_i}$, $\mathbf{B}_{m_i}$ and $\mathbf{C}_{m_i}$, $i = 1, 2$, $(D_{m_i} = 0,\ i = 1, 2)$ where $\mathbf{C}_{m_i} = [0 \ldots 0\ 1]$, $i = 1, 2$.

Eq. 7 is now written in the state-space as given next:

$$\begin{aligned}
\mathbf{x}^0_{m_1}(k+1) &= \mathbf{A}_{m_1}\mathbf{x}^0_{m_1}(k) + \mathbf{B}_{m_1}u(k), \quad y^0_{m_1}(k) = \mathbf{C}_{m_1}\mathbf{x}^0_{m_1} \\
\mathbf{x}^0_{m_2}(k+1) &= \mathbf{A}_{m_2}\mathbf{x}^0_{m_2}(k) + \mathbf{B}_{m_2}y^0_p(k), \quad y^0_{m_2}(k) = \mathbf{C}_{m_2}\mathbf{x}^0_{m_2} \\
y^0_m(k) &= y^0_{m_1}(k) + y^0_{m_2}(k)
\end{aligned} \tag{9}$$

where $y^0_{m_1}(k)$ and $y^0_{m_2}(k)$ stand for the outputs of submodels transfer functions G_{m_1} and G_{m_2} and $\mathbf{x}^0_{m_1}(k)$ and $\mathbf{x}^0_{m_2}(k)$ stand for the submodels states.

The H-step-ahead prediction of undelayed process model in the state-space form is, according to Eq. 9, written as follows:

$$\begin{aligned}
y^0_m(k+H) &= y^0_{m_1}(k+H) + y^0_{m_2}(k+H) \\
y^0_{m_1}(k+H) &= \mathbf{C}_{m_1}\left(\mathbf{A}^H_{m_1}\mathbf{x}^0_{m_1}(k) + \Sigma\mathbf{B}_{\mathbf{m_1}}u(k)\right) \\
y^0_{m_2}(k+H) &= \mathbf{C}_{m_2}\left(\mathbf{A}^H_{m_2}\mathbf{x}^0_{m_2}(k) + \Gamma_1\mathbf{B}_{\mathbf{m_2}}w(k) - \Gamma_2\mathbf{B}_{m_2}e(k)\right)
\end{aligned} \tag{10}$$

where the matrices Σ, Γ_1 and Γ_2 are

$$\begin{aligned}
\Sigma &= \left(\mathbf{A}^H_{m_1} - \mathbf{I}_1\right)\left(\mathbf{A}_{m_1} - \mathbf{I}_1\right)^{-1} \\
\Gamma_1 &= \left(\mathbf{A}^H_{m_2} - \mathbf{I}_2\right)\left(\mathbf{A}_{m_2} - \mathbf{I}_2\right)^{-1} \\
\Gamma_2 &= \left(\mathbf{A}^H_{m_2} - a_r^H\mathbf{I}_2\right)\left(\mathbf{A}_{m_2} - a_r\mathbf{I}_2\right)^{-1}
\end{aligned} \tag{11}$$

and where $\mathbf{I}_1$ and $\mathbf{I}_2$ stand for unity matrices of an appropriate dimension. Eq. 10 is obtained by assumption of a constant input variable in the whole prediction horizon $(u(k) = u(k+1) = \ldots = u(k+H-1))$, and assuming the exponentially decreasing control error which can be defined by the following equation

$$y^0_p(k+i) = w(k+i) - a_r^i e^0(k),\ i = 1, \ldots, H$$

where $e^0(k) = w(k) - y^0_p(k)$. The exponential factor a_r $(0 < a_r < 1)$, which introduces an exponentially decreasing estimated control error defines the behavior of the closed-loop system, as follows from [17].

The main idea of the predictive functional control is given by the following equation where the exponential decreasing of control error is introduced:

$$w(k+H) - y^0_m(k+H) = a_r^H \cdot \left(w(k) - y^0_p(k)\right) \tag{12}$$

where $w(k)$ stands for the current reference signal, $y^0_p(k)$ is the undelayed process output signal and $y^0_m(k+H)$ is the H-step-ahead prediction of the undelayed process output.

By introducing the model-based prediction of the output from Eqs. 10, 11 to 12, the control law of the decomposed-model predictive control becomes

$$u(k) = g\left(w(k) - y_p^0(k)\right) + \mathbf{K}_{m_1}\mathbf{x}_{m_1}^0(k) + \mathbf{K}_{m_2}\mathbf{x}_{m_2}^0(k) + \mathbf{K}_{y_p} y_p^0(k) \tag{13}$$

and where

$$\begin{aligned} g_0 &= \mathbf{C}_{m_1}\left(\mathbf{A}_{m_1}^H - I_1\right)\left(\mathbf{A}_{m_1} - \mathbf{I}_1\right)^{-1}\mathbf{B}_{m_1} \\ g &= g_0^{-1}\left(1 - a_r^H + \mathbf{C}_{m_2}\left(\Gamma_2 - \Gamma_1\right)\mathbf{B}_{m_2}\right) \\ \mathbf{K}_{m_1} &= g_0^{-1}\mathbf{C}_{m_1}\left(\mathbf{I}_1 - \mathbf{A}_{m_1}^H\right) \\ \mathbf{K}_{m_2} &= g_0^{-1}C_{m_2}\left(\mathbf{I}_2 - A_{m_2}^H\right) \\ \mathbf{K}_{y_p} &= g_0^{-1}\mathbf{C}_{m_2}\left(\mathbf{I}_2 - \mathbf{A}_{m_2}^H\right)\left(\mathbf{A}_{m_2} - \mathbf{I}_2\right)^{-1}\mathbf{B}_{m_2} \end{aligned} \tag{14}$$

The control law of CFMPC can be now written in a simplified form as

$$u(k) = g\left(w(k) - y_p^0(k)\right) + F(z)u(k) + H(z)y_p^0(k) \tag{15}$$

where

$$\begin{aligned} F(z) &= \mathbf{K}_{m_1}\left(z\mathbf{I}_1 - \mathbf{A}_{m_1}\right)^{-1}\mathbf{B}_{m_1} \\ H(z) &= \mathbf{K}_{m_2}\left(z\mathbf{I}_2 - \mathbf{A}_{m_2}\right)^{-1}\mathbf{B}_{m_2} + \mathbf{K}_{y_p} \end{aligned} \tag{16}$$

The variable $y_p^0(k)$ cannot be measured, but it can be estimated from the available signals of the process model and the current process output using a well-known Smith predictor paradigm:

$$y_p^0(k) = y_p(k) - y_m(k) + y_m^0(k) \tag{17}$$

The control scheme of CFMPC is presented in Fig. 2.

A very important feature of all control algorithms is their behavior at low frequencies. The control algorithm should be able to suppress the control error in the steady-state, i.e., the control law should have an integral nature. The control law of CFMPC in the $\mathcal{Z}$*-domain* is written as:

$$U(z) = (1 - F(z))^{-1}\left(gE^0(z) + H(z)Y_p^0(z)\right) = G_e(z)E^0(z) + G_{y_p}(z)Y_p^0(z) \tag{18}$$

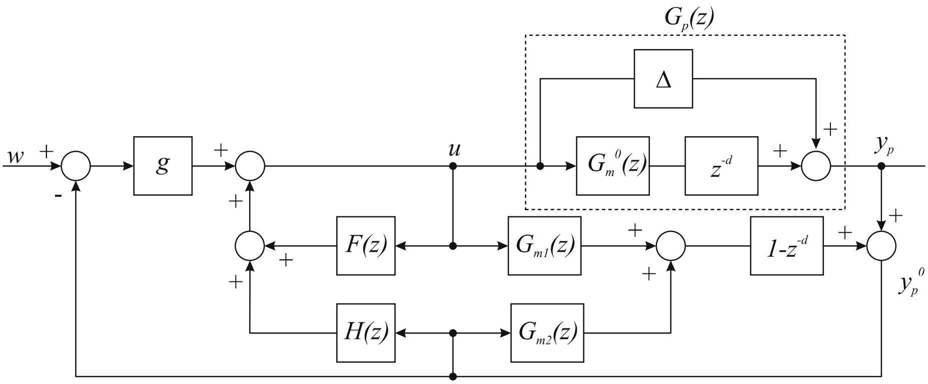


Fig. 2 The control scheme of CFMPC

where $Y_p^0(z)$ and $E^0(z)$ stand for the $\mathcal{Z}$*-transforms* of the estimated output variable $y_p^0(k)$ and the undelayed control error $e^0(k)$, respectively.

To prove the integral nature we have to investigate the behavior of the control law at low frequencies, i.e., we have to show that the transfer function between the control error and the control variable ($G_e(z)$) has a pole at $z = 1$. By calculating the denominator of the transfer functions $G_e(z)$ in Eq. 18 at $z = 1$ and by taking into account Eq. 14 and Eq. 16 the following is obtained:

$$1 - F(1) = 1 - g_0^{-1}\mathbf{C}_{m_1}\left(\mathbf{I} - \mathbf{A}_{m_1}^H\right)\left(\mathbf{I} - \mathbf{A}_{m_1}\right)^{-1}\mathbf{B}_{m_1} = 0 \tag{19}$$

This means that

$$G_e(1) = \frac{g}{1 - F(1)} \to \infty$$

and this proves that the control law of CFMPC is indeed integral in nature, i.e., y_p asymptotically tracks a step reference signal.

3 Robust Stability of CFMPC

The performance and robustness of model-based control schemes depend mainly on the uncertainty between the model and the plant. In the case of a stabilization problem, the most important factor is the robust stability of the closed-loop system. One of the most important tools for investigating the stability of the system in the presence of model-plant uncertainty is the norm-bounded small-gain theorem, as discussed by [3]. The small-gain theorem provides a sufficient condition for the stability of the control system. This means that a violation of the small-gain stability criteria, even with an exact knowledge of the uncertainty, may or may not lead to instability [1].

Additive perturbation, coprime-factor perturbation, multiplicative perturbation at the control input and multiplicative perturbation at the control output, are the main forms of unstructured uncertainties in linear systems. In fact all these uncertainties can be presented by additive perturbations, as shown by [18].

Therefore, in our case the uncertainty between the actual plant $G_p(z)$ and the plant model $G_m(z)$ will be described by an additive unstructured uncertainty

$$G_p(z) = G_m(z) + \Delta(z) \tag{20}$$

The structure of $\Delta(z)$ is unknown but stable, and it is an upper-bounded function in the frequency domain

$$|\Delta(e^{j\omega T})| < \delta(\omega), \quad \forall \quad \omega T \in [0, \pi] \tag{21}$$

where T stands for the sampling time. The upper bound $\delta(\omega)$ can be approximated from experiment. Taking into account Eq. 20 and Eq. 21 the family of plants $\mathcal{G}_a$ can be described by

$$\mathcal{G}_a = \{G_p \;:\; |G_p(e^{j\omega T}) - G_m(e^{j\omega T})| < \delta(\omega), \quad \forall \quad \omega T \in [0, \pi]\} \tag{22}$$

In Fig. 2 the coprime-factorized model predictive control scheme is presented, where Δ represents the additive unstructured uncertainty and g, $F(z)$ and $H(z)$ stand

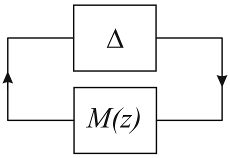


Fig. 3 The interconnection structure

for the controller that stabilizes the nominal plant. The controller can also be denoted by the triplet (g, F, H). A rearrangement of the system in Fig. 2, where all the external inputs and outputs are neglected, results in a general M-Δ interconnection structure (Fig. 3), as defined by [9]. The interconnection matrix M derived from Fig. 2 equals

$$M(z) = \frac{-g + H(z)}{1 - F(z) - (-g + H(z))\, G_m(z) - \left((1 - F(z))\, G_{m_1}(z) + (-g + H(z))\, G_{m_2}(z)\right)\left(1 - z^{-d}\right)} \tag{23}$$

Lemma 1 (Robust stability and the robust-stability margin for additive perturbation) *Assuming that all plants $G_p(z)$ in the family $\mathcal{G}_a$ defined in Eq. 22 have the same number of unstable poles (outside the unity circle in the z-plane) and that the controller*

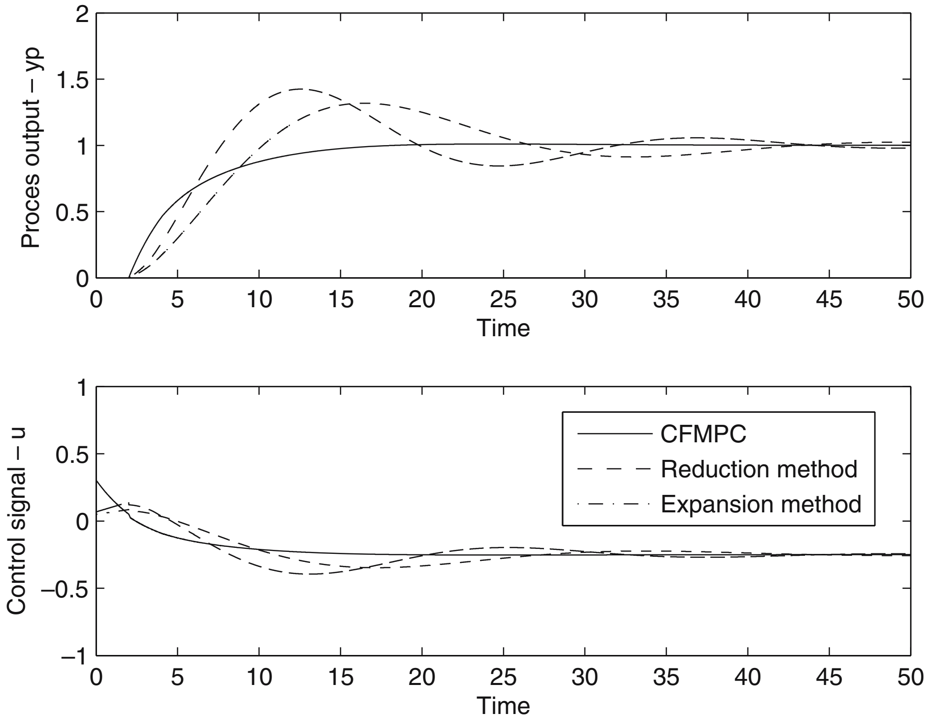


Fig. 4 Closed-loop responses of the output signal for a unit step change of the reference signal

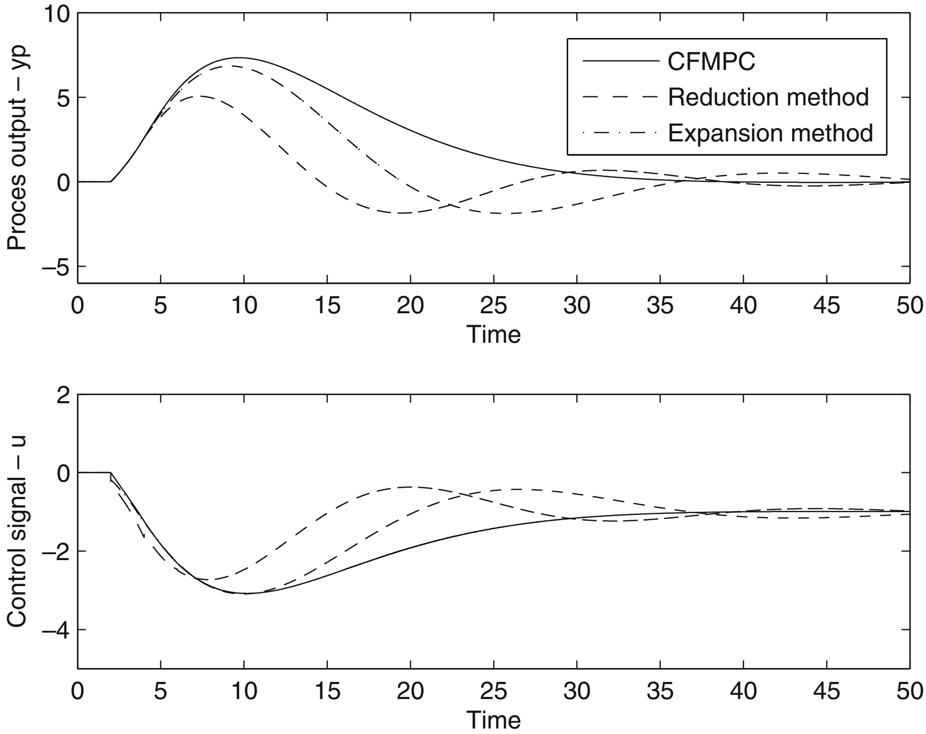


Fig. 5 Closed-loop responses of the output signal for a unit step input disturbance

(g, F, H) internally stabilizes the nominal plant G_m, then the system is robustly stable if and only if

$$\sup_{\omega} |M(e^{j\omega T})\delta(\omega)| < 1 \tag{24}$$

If we define the upper robust-stability margin $\overline{\delta}(\omega)$ as

$$\overline{\delta}(\omega) \triangleq \frac{1}{|M(e^{j\omega T})|}, \quad \omega T \in [0, \pi] \tag{25}$$

then the system is robustly stable if and only if

$$|\delta(\omega)| < \overline{\delta}(\omega), \quad \forall\, \omega T \in [0, \pi] \tag{26}$$

Lemma 1 is the result of applying the small-gain theorem to the interconnection structure from Fig. 3. Taking into account Lemma 1 and Eq. 23 the following theorem for the robust stability of CFMPC is obtained.

Theorem 1 (Robust stability of CFMPC for additive perturbation) *Assuming that all plants $G_p(z)$ in the family $\mathcal{G}_a$*

$$\mathcal{G}_a = \{G_p : \ |G_p(e^{j\omega T}) - G_m(e^{j\omega T})| < \delta(\omega), \ \forall\, \omega T \in [0, \pi]\} \tag{27}$$

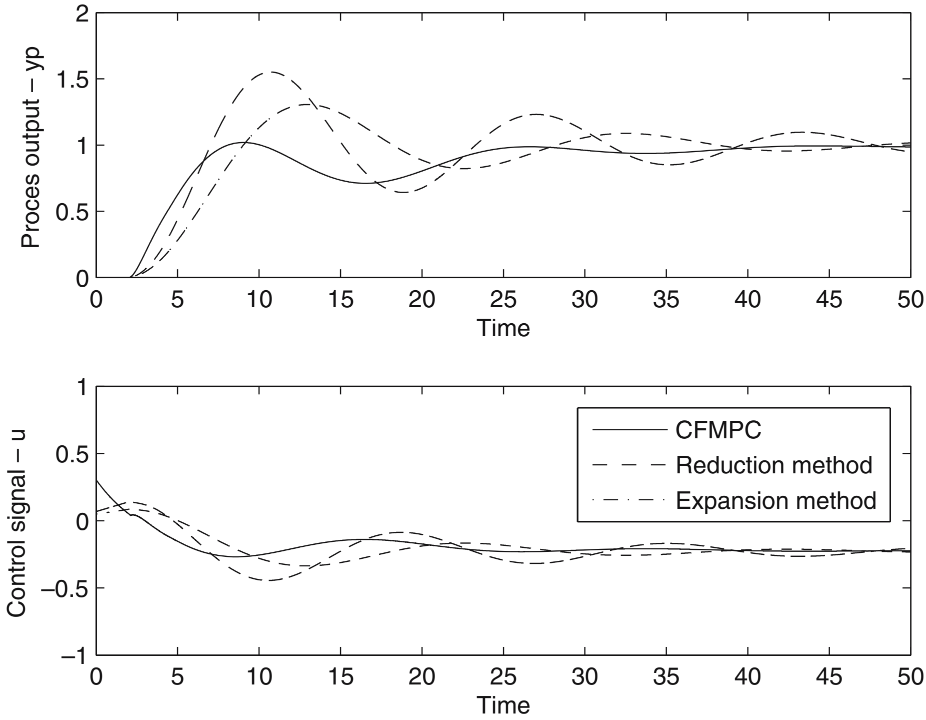


Fig. 6 Closed-loop responses of the output signal for a unit step change of the reference signal

have the same number of unstable poles (outside the unity circle in the z-plane) and that the controller (g, F, H) internally stabilizes the nominal plant G_m, then the system is robustly stable if and only if

$$\left|\frac{-g+H(z)}{1-F(z)-(-g+H(z))\,G_m(z)-\left((1-F(z))\,G_{m_1}(z)+(-g+H(z))\,G_{m_2}(z)\right)\left(1-e^{-jd\omega T}\right)}\delta(z)\right|<1 \tag{28}$$

where $z = e^{j\omega T}$.

Remark 1 (Robust upper-stability margin of CFMPC for additive perturbation) The robust upper-stability margin $\overline{\delta}_a(\omega)$ of CFMPC for additive unstructured perturbations is defined as follows:

$$\overline{\delta}(\omega) \triangleq \left|\frac{1-F(z)-(-g+H(z))\,G_m(z)-\left((1-F(z))\,G_{m_1}(z)+(-g+H(z))\,G_{m_2}(z)\right)\left(1-e^{-jd\omega T}\right)}{-g+H(z)}\right| \quad \omega T\in[0,\pi] \tag{29}$$

where $z = e^{j\omega T}$.

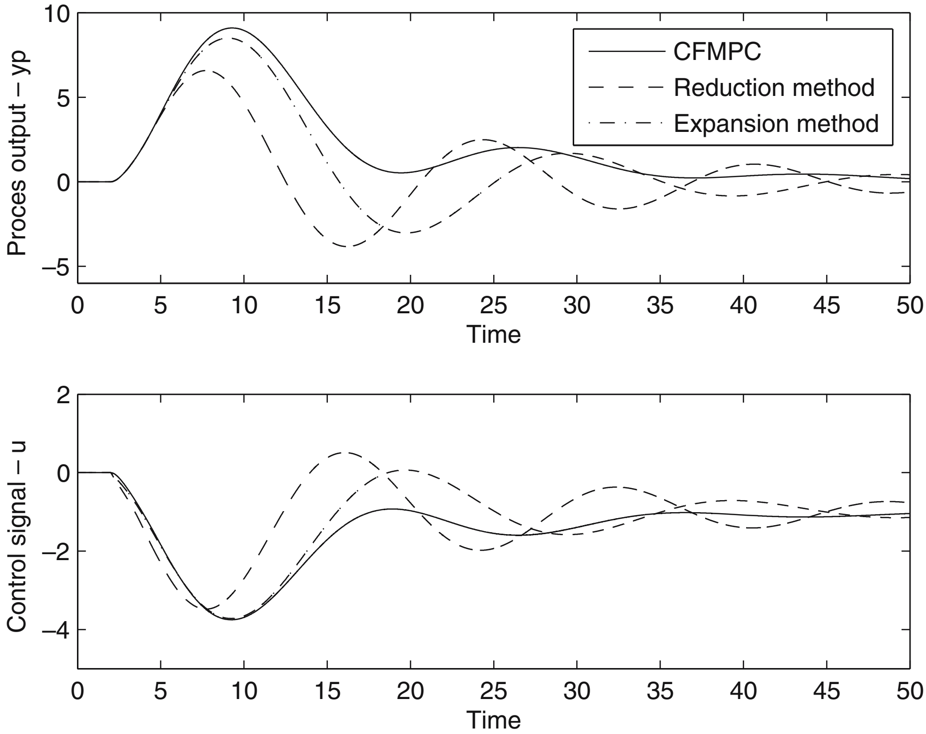


Fig. 7 Closed-loop responses of the output signal for a unite step input disturbance

According to the robust upper-stability margin the system is robustly stable if and only if

$$|\delta(\omega)| < \overline{\delta}(\omega), \quad \forall\, \omega T \in [0, \pi] \tag{30}$$

4 Simulation Study

In the simulation study the same test processes were used as those shown by [12], where the enhanced PID control strategy is compared with the results of different authors. This comparison shows that the enhanced PID control strategy [12] in two different modifications, gives better results than the others. This was the reason to make the comparison between the modified enhanced PID control and CFMPC for the unstable processes with a time delay.

4.1 Example

The observed plant is written in the form of a discrete-time transfer function, as given below:

$$G_p(z) = \frac{0.01}{z - 1.0025} z^{-200} \tag{31}$$

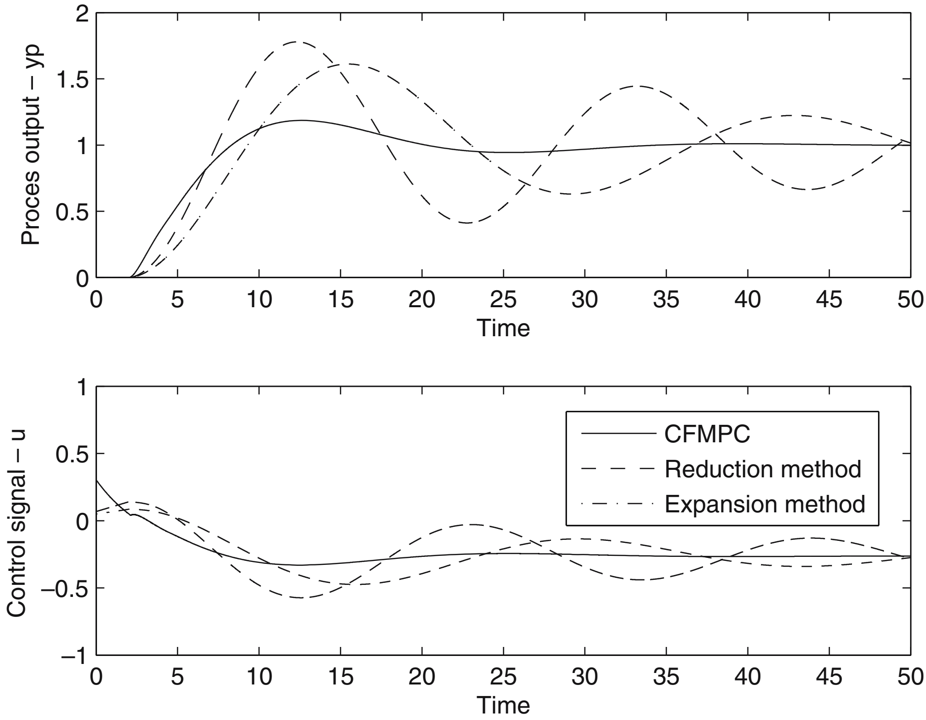


Fig. 8 Closed-loop responses of the output signal for a unit step change of the reference signal

with the sampling time $T_s = 0.01s$. Assuming the perfect modeling of the process, i.e., $G_m(z) = G_p(z)$ the control results using Park's tuning methods and the proposed CFMPC method are shown in Fig. 4, and in Fig. 5 where the proposed CFMPC method is compared with Park's method with the model reduction technique and with the Taylor-series expansion. The comparison is made for the reference tracking mode and the disturbance rejection mode with the same set of parameters. The CFMPC tuning parameters are the following: the coincidence horizon is chosen to be greater than the dead-time of the process model $(H > d)$ $H = 250$, the time constant of the reference model is $a_r = 0.9950$ and the design polynomial $\mathcal{A}(z) = z - 0.9995$, i.e., $\alpha_1 = 0.9995$ which is chosen according to the robust-stability study of the closed-loop system. The process model is factorized into coprime factors: $G_{m_1}(z) = \frac{0.01}{z-0.9995}$ and $G_{m_2} = \frac{0.003}{z-0.9995}$. The CFMPC controller is then given by: $g = 0.3035$, $F(z) = \frac{0.0005}{z-0.9995}$ and $H(z) = \frac{-0.3z+0.3}{z-0.9995}$. It can be seen that the results of CFMPC are much better when compared to the results obtained by Park's method in the case of reference tracking, but the disturbance rejection results are very similar.

Let us now add to the plant given in Eq. 31 a parasitic stable pole, an uncertainty in the time delay, an uncertainty at the unstable pole and an uncertainty in the plant gain. The plant gain is higher than the model gain. The plant transfer function is then the following

$$G_p(z) = \frac{(0.0001245z + 0.0001237)}{z^2 - 1.981z + 0.9805} z^{-200} \tag{32}$$

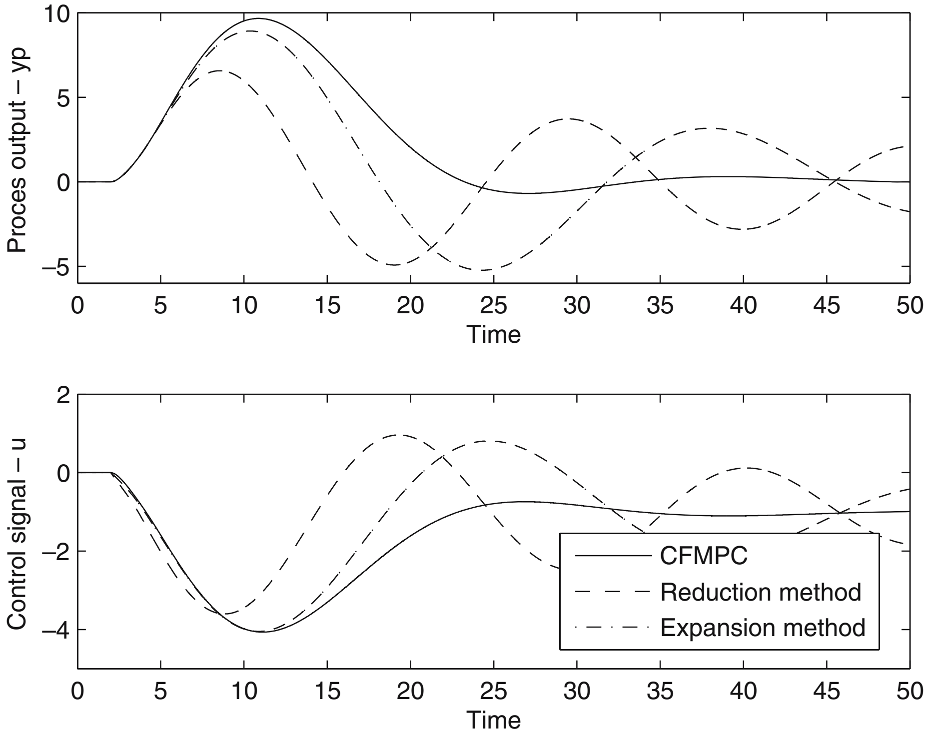


Fig. 9 Closed-loop responses of the output signal for a unite step input disturbance

The plant from Eq. 32 is by using Park's identification method [12] modeled by the model transfer function

$$G_m(z) = \frac{0.01}{z - 1.0025} z^{-210} \tag{33}$$

The uncertainty between the plant in Eq. 32 and the model in Eq. 33 is denoted as $\Delta_1(z)$ and presented in Fig. 10 as $|\Delta_1(e^{j\omega T_s})|$. In Fig. 10 we can seen that all the observed algorithms result in robustly stable control for the proposed type of uncertainty. The comparisons of all the control results are given in Fig. 6 and in Fig. 7. It can be seen that the response of the CFMPC gives much better results, both in the case of the reference tracking and also in the case of the disturbance rejection.

The same experiment was also done for the case when the plant gain is lower than the model gain. The process transfer function is the following

$$G_p(z) = \frac{(0.0001076z + 0.0001069)}{z^2 - 1.981z + 0.9805} z^{-200} \tag{34}$$

The uncertainty between the process and the model is denoted as $\Delta_2(z)$, and is presented in Fig. 10 as $|\Delta_2(e^{j\omega T_s})|$. The comparisons of all the control results are

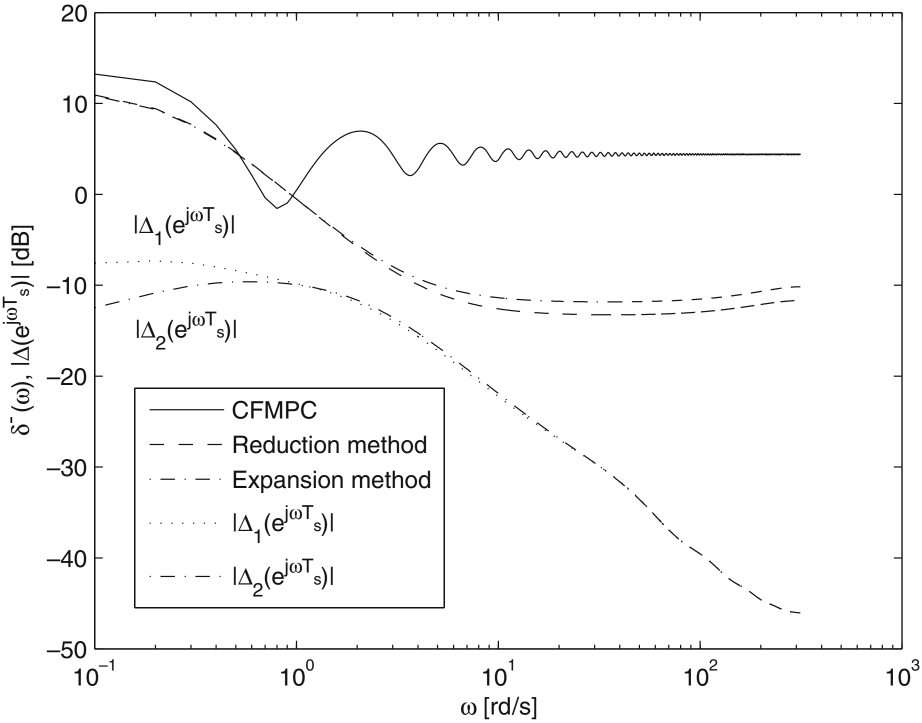


Fig. 10 Comparison of robust stability for all three approaches

given in Fig. 8 and in Fig. 9. The reference tracking and the disturbance rejection in the case of CFMPC is in this case even better, compared with the other two methods. And in Fig. 10 the robust stability of the proposed control algorithm is shown for this uncertainty. It can be seen that the proposed CFMPC control algorithm gives higher robust-stability margin than those given in [12].

5 Conclusion

In this paper the coprime-factorized model predictive control is given for unstable processes with a time delay. The proposed control approach is not limited to this type of process, but it can be implemented on a wide range of different processes. The main advantages of the proposed approach is the very simple design, which results in better performance than obtained before. To show the potential of the proposed approach, a comparison was made with some of the most promising results known from the literature, and it can be concluded that by using this new approach, even better performance can be obtained.

Acknowledgements The work was done in the frame of a bilateral project between France and Slovenia, Predictive control, FR-2002-1. The author would like to thank to Egide, France and Ministry of Education, Science and Sport, Slovenia, for the support.

Appendix

If the system given by the transfer function in Eq. 35, which describes the relation between the input, $U(z)$, and the output of the system, $Y(z)$, is observable,

$$H(z) = \frac{Y(z)}{U(z)} = \frac{b_1 z^{n-1} + b_2 z^{n-2} + \cdots + b_n}{z^n + a_1 z^{n-1} + \cdots + a_n} \tag{35}$$

then it can be transformed in to the observable canonical state-space form denoted by the matrices triple **A**, **B** and **C** as follows:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ y(k) &= \mathbf{C}\mathbf{x} \end{aligned} \tag{36}$$

where $\mathbf{x}(k)$ denotes the states of the system in the discrete-time domain, $u(k)$ and $y(k)$ are the input and the output in discrete time and where

$$\mathbf{A} = \begin{bmatrix} 0\ 0 \cdots 0 & -a_n \\ 1\ 0 \cdots 0 & -a_{n-1} \\ 0\ 1 \cdots 0 & -a_{n-2} \\ \vdots \quad \ddots & \vdots \\ 0\ 0 \cdots 1 & -a_1 \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} b_n \\ b_{n-1} \\ b_{n-2} \\ \vdots \\ b_1 \end{bmatrix},\ \mathbf{C} = \begin{bmatrix} 0\ 0 \cdots 0\ 1 \end{bmatrix} \tag{37}$$

Using the state-space notation, the transfer function $H(z)$ is given as

$$H(z) = \mathbf{C}\left(z\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B} \tag{38}$$

where **I** stands for unity matrix of appropriate dimension.

References

1. Banerjee, P., Shah, S.L.: The role of signal processing methods in the robust design of predictive control. Automatica **31**(5), 681–695 (1995)
2. DePaor, A.M.: A modified smith predictor and controller for unstable processes with time delay. Int. J. Control **41**, 1025–1036 (1985)
3. Doyle, J., Francis, B., Tannenbaum, A.: Feedback Control Theory. Macmillan, New York (1990)
4. Glover, K., McFarlane, D.: Robust stabilization of normalized coprime factor plant descriptions with h_∞-bounded uncertainty. IEEE Trans. Automat. Contr. **34**(8), 821–830 (1989)
5. Jacob, E.F., Chidambaram, M.: Design of controllers for unstable first-order plus time delay systems. Comput. Chem. Eng. **20**(5), 579–584 (1996)
6. Kaya, I.: Imc based automatic tuning method for pid controllers in a smith predictor configuration. Comput. Chem. Eng. **28**, 281–290 (2004)
7. Lee, Y., Lee, J., Park, S.: Pid controller tuning for integrating and unstable processes with time delay. Chem. Eng. Sci. **55**, 3481–3493 (2000)
8. Maciejowski, J.M.: Predictive Control with Constraints. Prentice-Hall, Pearson Education Limited, Harlow, UK (2002)
9. Morari, M., Zafiriou, E.: Robust Process Control. Prentice-Hall, Englewoods Cliffs, NJ (1989)
10. Nagrath, D., Prasad, V., Bequette, B.W.: A model predictive formulation for open-loop unstable cascade systems. Chem. Eng. Sci. **57**, 365–378 (2002)
11. Park, J.H., Sung, S.W., Lee, I.-B.: Improved relay autotuning with static load disturbance. Automatica **33**(3), 711–715 (1997)
12. Park, J.H., Sung, S.W., Lee, I.B.: An enhanced pid control strategy for unstable processes. Automatica (1998)
13. Rotstein, G.E., Lewin, D.R.: Simple pi and pid tuning for open-loop unstable systems. Ind. Eng. Chem. Res. **30**, 1864– 1869 (1991)

14. Shafiei, Z., Shenton, A.T.: Frequency-domain design of pid controllers for stable and unstable systems with time delay. Automatica **33**(12), 2233–2232 (1997)
15. Tan, W., Marquez, H.J., Chen, T.: Imc design for unstable processes with time delays. J. Process Control **13**, 203–213 (2003)
16. Venkatashankar, V., Chidambaram, M.: Design p and pi controllers for unstable first-order plus time-delay systems. Int. J. Control **60**, 137–144 (1994)
17. Škrjanc, I., Matko, D.: Predictive functional control based on fuzzy model for heat-exchanger pilot plant. IEEE Trans. Fuzzy Syst. **8**(6), 705–712 (2000)
18. Wang, Y.: On robust stabilization regions of unstructured uncertainties. Syst. Control Lett. **30**, 13–17 (1997)